Get Your Integral On: Solutions

1) $\int_1^e \frac{\sqrt{ln(x)}}{x}\,dx = \frac{2}{3}$, u-substitution

2) $\int \frac{dx}{\sqrt{9-x^2}} = sin^{-1}(\frac{x}{3}) + c$, trigonometric substitution

3) $\int xcos^3(x)\,dx = \frac{1}{9}(3xsin(x)(3 - sin^3(x)) + 6cos(x) + cos^3(x)$ + c, integration by parts

4) $\int \frac{ln(x)}{x^2}\,dx = -\frac{ln(x)+1}{x} + c$, integration by parts

5) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2e^{\sqrt{x}} + c$ substitution

6) $\int \sin^3(x) + \sin(x)cos^2(x)\,dx = -\cos(x) + c$; use $\sin^2(x) + \cos^2(x) = 1$

7) $\int_{-1}^0 \frac{x^3+5x^2+12x+19}{x^2+4x+4}\,dx = [\frac{x^2}{2} + x - \frac{7}{x+2} + 4\ln(x+2) + 2]_{-1}^0 = 4 + \ln(16)$
Since the degree of the numerator is bigger, start with a long division. Integrate the remainder with partial fractions.

8) $\int \frac{dx}{9-x^2} = \frac{1}{6}ln|\frac{x+3}{x-3}| + c$. Consult formula #42 in your integral tables.

9) $\int \frac{dx}{\sqrt{x}-\sqrt[4]{x}} = 2\sqrt{x} + 4x^{1/4} + 4\ln(x^{1/4} - 1) + c$; begin with $u = x^{1/4}, dx = 4u^3du$, complete integral using long division.

10) $\int \frac{ln(x)}{x^2}\,dx = -\frac{ln(x)+1}{x} + c$ integrate by parts

11) $\int_1^3 \frac{dx}{x^2-4x+4}$ integral does not converge; discontinuity at x=2.
This handout was distributed before we covered improper integrals, so it's OK if you did not notice this discontinuity. However, keep a sharper eye out during the test.

12) $\int xcos^3x\,dx = \frac{1}{9}(3x\sin(x)(3 - \sin^2(x)) + 6\cos(x) + \cos^3(x)) + c$; use trig identities and integration by parts.

13) $\int e^{\sqrt{x}}\,dx = 2e^{\sqrt{x}}(\sqrt{x} - 1) + c; u = \sqrt{x}, dx = 2udu$, integrate $2\int e^u u\,du$ by parts

14) $\int \frac{dx}{x^3-x} = \frac{1}{2}ln|x^2 - 1| - ln|x| + c$, partial fractions

15) $\int sin(x)cos(x)e^{cos(2x)}\,dx = -\frac{1}{4}e^{cos(2x)} + c$ u-sub: $u = cos(2x)$

16) $\int \sqrt{4-x^2}\,dx = \frac{1}{2}\sqrt{4-x^2}x + 2sin^(-1)(\frac{x}{2}) + c$

17) $\int xln(x^2+1)\,dx = \frac{1}{2}((x^2+1)\ln(x^2+1) - x^2) + c$ u-sub, $u = x^2 + 1$

18) $\int \frac{sin(1/x)}{x^2}\,dx = cos(\frac{1}{x}) + c$, u-sub $u = \frac{1}{x}$

19) $\int_0^\pi sin^3(x)cos(x)\,dx = 0$ u-sub, $u = sin(x)$

20) $\int \frac{cos(ln(x))}{x}\,dx = sin(ln(x)) + c, u = ln(x)$

21) $\int_0^3 te^{-t}\,dx = [-e^{-t}(t+1)]_0^3 = 1 - \frac{4}{e^3}$ integrate by parts

22) $\int_0^{\sqrt{2}/2} \frac{x^3}{\sqrt{1-x^2}}\, dx = \frac{1}{12}(8 - 5\sqrt{(}2))$ trigonometric substitution

23) $\int \frac{dy}{y(k-y)}$ where k is a constant $= \frac{1}{k}(ln|\frac{y}{k-y}| + c$

24) $\int \frac{x^3}{(1+9x^4)^{3/2}}\, dx = -\frac{1}{18\sqrt{9x^4+1}} + c$

25) $\int_0^4 \frac{2t}{9+t^2}\, dt = [ln(t^2+9)]_0^4 = ln(\frac{25}{9})$

26) $\int_0^\pi cos(3x)cos(3sin(3x))\, dx = 0$ u-substitution with $u = 3sin(3x)$

27) $\int \frac{x^2+2}{(x^2+1)(x-1)} dx = \frac{1}{4}(-ln(x^2+1) + 6ln(x-1) - 2tan^{-1}(x)) + c$

If you have more time to do integrals, see page 509-510 of your textbook; Assorted Integrations, and Additional and Advanced Exercises